Differentiating sine / cosine

The sine / cosine functions differentiate using standard sets of rules:

$$\frac{d}{dt}(\sin t) = \cos t \qquad \frac{d}{dt}(\cos t) = -\sin t$$

$$\frac{d}{dt}(-\sin t) = -\cos t \qquad \frac{d}{dt}(-\cos t) = \sin t$$

Slightly more complicated is sin (ωt) or sin ($\omega t + \phi$) etc. In these cases the expression within the sine or cosine is itself differentiated w.r.t the variable "t", in addition to the sin or cos function itself: these are then multiplied together.

$$\frac{d}{dt}(A \sin \omega t) = \omega A \cos \omega t \quad \text{and} \quad \frac{d}{dt}(A \cos \omega t) = -\omega A \sin \omega t \qquad \boxed{\text{since } \frac{d}{dt}(\omega t) = \omega}$$

For a further example, if the sin / cos function is now a more complicated function of *t* say, notice how we still differentiate it (as well as the sin / cos itself) ; e.g. for sin (kt^2) and cos (kt^3) etc.

$$\frac{d}{dt}\left[\sin(kt^2)\right] = 2kt.\cos(kt^2) \qquad \text{and} \qquad \frac{d}{dt}\left[\cos(kt^3)\right] = -3kt^2.\sin kt^3$$

If the function has two variables in it, (as with some of the functions in our lecture course, e.g. $y = y_0 \sin(\omega t - kx)$), then the differentiation works in the same way;

$$\frac{d}{dt}(y) = \omega y_0 \cos(\omega t - kx) \quad \text{and} \quad \frac{d}{dx}(y) = -ky_0 \cos(\omega t - kx)$$

In general:
$$\frac{d}{dx}[f(g(x))] = \frac{df}{dx} \times \frac{dg}{dx}$$

This means we differentiate the outside function, leave the argument of the outside function alone, and then multiply by the derivative of the inside function. For example;

$$\frac{d}{dx}[\cos(3x^2 + 1)] = \frac{df}{dx} \times \frac{dg}{dx} = -\sin(3x^2 + 1) \times 6x = -6x.\sin(3x^2 + 1)$$

Practice Problems:

Complete the following (see PV or your tutor for solutions):

$$\frac{d}{dt}\left[A\omega \sin(\omega t)\right] =$$

$$\frac{d}{dx}\left[A \cos(\omega t - kx)\right] =$$

$$\frac{d}{dt}\left[A\omega \cos(\omega t - kx)\right] =$$

$$\frac{d}{dt}\left[\frac{-A\cos(\omega t - kx)}{\omega}\right] =$$

$$\frac{d}{dk}\left(\frac{c.\sin(ka/2)}{(a/2)}\right) =$$